Bayes class 6

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# Heteroscedasticity

There are many types of heteroscedasticity. In most undergraduate statistics programmes, students are taught to watch out for some of these, but seldom taught how to work with them. This is a serious shortcoming as understanding uncertainty is at the heart of what makes a good statistician. Furthermore, predictions from an unstable system are bound to lead to trouble.

## Systematic heteroscedasticity

The first case is where we have $Var\left(X\right)=f\left(E\left(X\right)\right)$ or something similar, where the change in variance is directly related to a change in something observed. This can often be addressed with a transformation.

What is usually a good transformation when $Var\left(Y\right)∝E\left(Y\right)$?

What is usually a good transformation when $StdDev\left(Y\right)∝E\left(Y\right)$?

These transformations are special cases of what general transformation?

## Specific heteroscedasticity

The opposite case is when each observation has different variance with no discernible pattern. This case can also present as excess kurtosis or extreme values in one or more directions.

One approach used to address such issues is to adapt the residual distribution, usually replacing a non-robust but powerful distribution (*e.g.* normal) with a more robust alternative (*e.g.* Student-t). The result is that outlying observations become less influential, which is useful when modelling central tendencies.

Another approach is switching from a parametric model to a non-parametric model. This is somewhat similar to the above in that we abandon the usual assumptions (*e.g.* normal) and use a more free specification for the residuals.

What are the downsides to changing the assumed residual distribution?

## Class based heteroscedasticity

This is where groups/classes/treatments/levels of observations have naturally different variation.

The standard approach to addressing this is to model the groups/classes/treatments/levels separately.

What does a separate regression for each group imply about the regression lines?

What is the problem with doing a separate regression for each group?

How do we get one regression line with different variances for each group?

Simulate a sample of shoe sizes for boys and girls aged 9 to 18. You should have 5 for each age and gender, so 100 sizes in total in your sample. Both the sizes and their variances should depend on the age and gender. [Hint: a Poisson distribution with $λ=β\_{0}+β\_{1}Age+β\_{2}Gender$ would probably work (barely).]

Now run an ordinary regression and look at the diagnostic plots. Can you describe all the ways in which the resulting patterns differ from what is usually expected?

Now fit a model that accommodates these issues to some extent. Are the coefficients closer to the true values you selected?

**In all cases remember that it is the conditional variance that matters, not the raw variance!**